

LA CUARTA DIMENSIÓN

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OBJETIVO DEL CURSO:

El objetivo es presentar a los alumnos las ideas contemporáneas utilizadas en el estudio de las variedades lisas de dimensión 4.

Se cubrirán los aspectos topológicos de las descripciones de 'cubos con asas' y el cálculo de Kirby. Estas son en realidad la única manera de asegurar que dos variedades cerradas orientables lisas de dimensión 4 son difeomorfas.

REQUISITOS:

Haber cursado alguna materia del posgrado en la que se hayan cubierto los temas básicos acerca de variedades topológicas y diferenciables (haz tangente, cotangente, formas diferenciales).

Haber cursado una materia del posgrado en la que se hayan cubierto los temas básicos de topología algebraica como grupo fundamental, homología, cohomología.

TEMARIO:

Se seguirá el libro de Selman Akbulut, cuyo contenido es el siguiente:

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