

# Hyperbolic Conservation Laws

## Semestre 2024 - 2

### Maestría y Doctorado en Ciencias Matemáticas y de el Curso Avanzado de Ecuaciones Diferenciales (6 Credit Course)

**Professor:** Sarswati Shah

**Email:** sarswatishah@im.unam.mx

**Class schedule:**

- Monday, Tuesday 03:00 - 04:30 pm

**Hybrid Mode, In person or via Meet:**

Aula Teórica, Imate Unidad Juriquilla

**Office:** Cubículo 3 Imate-  
Juriquilla

**Phone :** 442-192-6283

Ext. 303

**Office hours:**

-By Appointment

**Google Meet:** The link will be provided later.

**Web Page:** <https://sarswatishah.github.io/>

**Reference textbooks:**

- - A. Bressan, Hyperbolic systems of conservation laws: The one-dimensional Cauchy problem, vol. 20 of Oxford Lecture Series in Mathematics and its Applications, Oxford University Press, Oxford, 2000.

- - T. Cazenave and A. Haraux, An Introduction to Semilinear Evolution Equations, Clarendon Press, Oxford, 1998.
- C. M. Dafermos, Hyperbolic conservation laws in continuum physics, vol. 325 of Grundlehren der Mathematischen Wissenschaften, Springer-Verlag, Berlin, fourth ed., 2016.
- - L. C. Evans, Partial Differential Equations, vol. 19 of Graduate Studies in Mathematics, American Mathematical Society, Providence, RI, second ed., 2010.
- - J. Smoller, Shock Waves and Reaction-Diffusion Equations, Springer-Verlag, New York, 1983.
- - G.B. Whitham, Linear and nonlinear waves. John Wiley & Sons, 2011.

**Prerequisites:** Basic knowledge of Partial Differential Equations (PDEs) and the solution of first order PDEs.

**Objective of course:**

- Analyzing the behavior of non-linear waves arising in many physical phenomena in the atmosphere and ocean.

**Topics:**

#### 1. Introduction to first order PDE

##### 1.1 Transport equation

- 1.2 Method of characteristics
- 1.3 Burgers equation and loss of regularity
- 1.4 Fully nonlinear equations
- 1.5 Introduction to Hamilton–Jacobi equations

## **2. Scalar conservation laws**

- 2.1 Integral (or weak) solutions
- 2.2 Non-uniqueness and Rankine–Hugoniot condition
- 2.3 Admissibility Conditions
- 2.4 Riemann’s problem
- 2.5 Riemann Invariant

## **3. Systems of conservation laws**

- 3.1 Integral solutions
- 3.2 Traveling waves and hyperbolic systems
- 3.3 Simple waves
- 3.4 Rarefaction waves
- 3.5 Shock waves and contact discontinuities

## **4. Global solutions to the Cauchy problem**

- 4.1 Front tracking approximate solution
- 4.2 Bounds on the total variation
- 4.3 Convergence to a limit solution

## **5. Continuous dependence on the initial data**

- 5.1 Unique solutions to the scalar conservation law (well posedness for the scalar Cauchy problem)
- 5.2 Linear Hyperbolic Systems
- 5.3 Nonlinear systems

**For Doubts:** If you have doubts or questions, you can discuss in office hours by request.