Polígonos de Newton, Poliedros y Geometría Tropical

Seminario de Geometría
Posgrado en Ciencias Matemáticas

Objetivo

Que los alumnos dominen las técnicas del polígono y poliedro de Newton para parametrizar curvas planas e hipersuperficies algebraicas y que entiendan las ideas detrás de sus extensiones al caso diferencial y la geometría tropical

Temario

La primera parte de cada tema estará dedicada al caso algebraico clásico y la segunda a la extensión del caso clásico a Ecuaciones diferenciales.

1. Curvas planas y ecuacines diferenciales ordinarias
   (a) El método del polígono de Newton-Puiseux para parametrizar curvas algebraicas planas [Cas2000, Pu1850]
   (b) extensión para encontrar soluciones de ecuaciones diferenciales ordinarias. [Fi1889, GS1991, Ca1993]
2. Hipersuperficies y ecuaciones en derivadas parciales.
   (a) El poliedro de Newton para parametrizar hipersuperficies algebraicas [McD1995],
   (b) Campos donde se calculan las parametrizaciones [Ra197, AI2009, AK2013]
   (c) Extensión a ecuaciones diferenciales parciales [AC2001,ARJ2003]
3. Codimensión arbitraria y sistemas de ecuaciones en derivadas parciales
   (a) La tropicalización de un ideal algebraico [MS2015]
   (b) uso de la tropicalización de un ideal para calcular parametrizaciones de variedades algebraicas de codimension arbitraria [AIL2010]
   (c) geometría tropical diferencial [Gr2017, AGT2016,AI2016, FGH20]

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